HW 4 SOLUTIONS

6.3. Draw Lorenz Curves and GINI coefficients – the first set of numbers represent incomes and the second set of numbers represent population. (SEE THE ASSOCIATED EXCEL WORKSHEET FOR SPECIFIC FORMULAS)

6.3.a. (100, 200, 300, 400) ; (50, 25, 75, 25)

in the first example total income is:

\[(50\times100) + (25\times200) + (75\times300) + (25\times400) = 42,500\]

and mean income is $42,500/175 which is 242.85

now to draw the Lorenz curve you need to find the income shares by quintiles

the share of income of the poorest 20% of the population

if there are a total of 175 people then 20% of them is 35 - so what share of total income does the poorest 35 people account for

well we know from the brackets that the poorest 50 earn $100 so the poorest 35 also earn $100 so the income share of the poorest 20% is now 35*(100) which is 3,500/42,500 this is about 8.2% - so the poorest 20% make 8.2% of income

now what about the poorest 40% of the population (70 people)

now we are dealing with 70 people we know that the first 35 make 3,500 now how much do the second poorest 35 make?

\[
\begin{align*}
$3,500 \\
+ 15\times$100 \\
+ 20\times$200 \\
\text{(the combined income for 15 of them is 15\times$100 which is$1,500 and the combined income for the remaining 20 is$200 each or$4,000)}
\end{align*}
\]

this is equal to $9,000. The income share is therefore equal to $9,000/42,500 which is about 21.2% - so the poorest 40% account for about 21.2% of income

the poorest 60% (105 people):

\[
\begin{align*}
$9,000 \\
+ 5\times200 \\
+ 30\times300 \\
\text{which is equal to$19,000. As a share of total income this is 19,000/42,500 or 44.7% of income}
\end{align*}
\]

the poorest 80% (140 people):

\[
\begin{align*}
$19,000 \\
+ 35\times300 \\
\text{which is equal to$29,500. As a share of total income this is 29,500/42,500 or 69.4% of income}
\end{align*}
\]

to check lets make sure we arrive at the total for 100% of the population (175 people)

$29,500
which is equal to $42,500 – or 100% of total income.

So our Lorenz curve can be drawn as

now what about the GINI coefficient:

use the formula (6.4) in the text where we are measuring the area in the figure above divided by the entire triangle – in our example this is equal to

\[
\frac{1}{2*175^2*242.9} \times \left[ (50*50(100-100) + 50*25(100-200) + 50*75(100-300) + 50*25(100-400)) + (25*50(200-100) + 25*25(200-200) + 25*75(200-300) + 25*25(200-400)) + (75*50(300-100) + 75*25(300-200) + 75*75(300-300) + 75*25(300-400)) + (25*50(400-100) + 25*25(400-200) + 25*75(400-300) + 25*25(400-400)) \right]
\]

solving for the GINI coefficient we have 0.2353

The formula for the coefficient of variation (standard deviation divided by the mean) is given in equation 6.3 of the text. Applying the formula we have 0.2446
6.3.b.) in the second distribution incomes double and population size remains constant – in this case the Lorenz curve, GINI coefficient, and coefficient of variation will be identical to part a.

6.3.c.) same thing as in part b. – except now the population size doubles – the Lorenz curve and gini coefficient will again be identical to case a.

6.3.d.) now population changes dramatically – and the Lorenz curve is shown relative to that for part a in the figure below.

![Lorenz Curve – (part d)](image)

it is clear from the figure that the GINI coefficient and coefficient of variation will be larger. Solving for actual values we have 0.2667 and 0.2876, respectively.

6.3.e.) in this part income is again equivalent to part a, however the share of the population with 300 increases while that for 200 and 400 falls. Below is the Lorenz curve relative again to part a. We immediately observe that this distribution is relatively more equal, consequently we again expect the GINI coefficient and the CV to be less relative to part A. Solving for both we have 0.2124 and 0.1894, respectively.
6.3.f.) again, income categories are equal to part a, however in this case there are greater numbers earning 200 and 400. As illustrated in the figure below, income distribution is slightly worse in the higher income categories relative to the benchmark case in part a. Again we expect the GINI coefficient to be slightly larger along with the coefficient of variation; solving for actual values we have 0.2528 and 0.2894.

7.5.a As development first begins, a nascent industrial sector that demands skilled labor comes into being. This will shift the demand curve for skilled labor to the right (you can draw a graph), so that the wage rate for skilled labor rises. However, the higher wages will draw more individuals into acquiring skills. This is a long-run effect, however, since skills aren’t acquired
overnight – but when it finally comes into play there are two implications. First the supply curve for skilled labor will move to the right, attenuating or even reversing the rise in skilled wage. Second, the supply curve for unskilled labor may actually decline and move to the left (if the rate of population growth is smaller than the rate of skill acquisition). This will bring up the unskilled wage (or at least slow its decline). Putting all this together, the net effect is first an increase, then a decrease, in the ratio of skilled to unskilled wages over the course of development.

7.5.b Read the Paukert and Ahluwalia research described in the chapter.

7.5.c This question asks you to re-study two things in the text: one, that an increase in inequality will lower average rates of savings if the savings function is concave (marginal savings rates decline with higher income) and therefore the average rate of growth. {make sure you understand this argument we discussed it thoroughly in class}. But second higher inequality may trigger a political demand for redistribution, and that such redistribution may take the form of taxation of increments to assets (rather than seizure and redistribution of assets themselves). This type of taxation can be especially bad for investment and growth as described in the text (why?).

7.5.d This question refers to the credit markets model. Recall how there is a minimum threshold of wealth (given a certain wage) below which access to entrepreneurial credit is impossible. If inequality increases, then more individuals fall below this minimum. Thus supply of entrepreneurs decreases. This is the same as shifting the labor demand curve in Figure 7.10 to the left. Likewise, the increased supply of laborers will shift the supply curve of labor to the right (see Figure 7.10). The net effect of all of this is that the equilibrium wage rate must decline.

7.7 The easiest way to do this question is to find out total income and total savings in an economy of say 100 people in the proportions given.

7.7a Given the percentages there will be 20 poor people, 50 middle class people, and 30 rich people. Total income will therefore be $500(20) + $2000(50) + $10,000(30) = $410,000 Total savings is $0(20) + $500(50) + $2000(30) = $85,000 The total savings rate is therefore $85,000/$410,000 or 20.7%. It is clear that overall savings is affected by income distribution (20 people save nothing at all).

7.7.b Yes clearly the overall savings will increase (20 more people save $500 each). Using the formula for the Harrod Domar model the rate of growth will increase.

7.7.c Consider a scenario where growth occurs by shifting people from the middle class to the rich. Because the rich save a lower fraction of income relative to the middle class, this will bring down the rate of savings over time and therefore the growth rate.

7.7.d Recap for your own understanding.

7.11 See also the discussion to 7.5.c. Recall that the credit market model in which for every wage rate, a minimum wealth size is required in order to obtain loans from the credit market. Recall that if the wage rate is lower, then business profits are higher. Therefore borrowers who
invest in businesses will be more willing to repay the loan, as they have more to lose in the process. It follows that if wage rates are lower, then the minimum wealth required to access the credit market goes down – it becomes easier to access the credit market.

We then used this to construct the supply curve of labor (laborers were those who could not raise money from the credit market to start a business of their own – they worked for those who could secure credit). Note that because of the reason in the previous paragraph, a higher wage rate induces greater supply of labor, simply because less people are now able to borrow from the credit market. This means that the supply curve of labor is upward sloping in the traditional way. By the same argument, the demand curve for labor is downward sloping. Their intersection determines the equilibrium wage rate.

The point now is to note well that the shapes and locations of these curves depend intimately on the going distribution of wealth in the economy. If the distribution worsens then at every wage rate, less people will be able to borrow from the credit market than before. Thus the supply curve of labor will shift out to the right, while the demand curve for labor will shift inwards to the left. The implication is that the equilibrium wage rate comes down.

8.1 This question should be answered based on a general understanding of the material covered in the text.