Solutions to HW #2  
Development Economics, Debraj Ray, #1, #3, #6, #8, #10  
(solutions are written by the author or slightly modified)

1.a The running costs include labor ($2000 times 100) and cotton fabric, which is $600,000. Thus total costs are $800,000 per year.

Total revenues are equal to price time quantity (q=100,000 and p=$10) thus revenues are $1 million.

Total profits, not counting setup investment, therefore are $200,000 per year.

1.b To figure out income generated, we must count profits and the wage payments to workers as well, which are $200,000 ($2000 * 100 workers). Thus income generated is wages plus profits (there are no rents here), which is $400,000 per year.

1.c The output of the firm is $1 million per year. The firm’s installed capital is $4 million. Therefore the capital-output ratio is 4. Notice that the capital equipment can be used over and over again (though it might depreciate over time). Therefore a capital-output ratio larger than one is perfectly compatible with the notion of profitability.

3.a Neglecting depreciation in this exercise, The Harrod-Domar model leads us to the equation: $g = \frac{s}{\theta}$, where $g$ is the aggregate growth rate, $s$ is the rate of savings, and $\theta$ is the capital-output ratio. Here $s = \frac{1}{5}$ and $\theta = 4$.

\[
\frac{s}{\theta} = \frac{1/5}{4} = \frac{1}{20}
\]

So $g = \frac{1}{20}$, or 5% per year.

3.b We know that the per-capita growth rate is the aggregate growth rate minus the population growth rate.

\[
\frac{s}{\theta} = g + n + \delta
\]

Therefore, if the required per-capita growth rate is 4% and the population growth rate is 3%, the required aggregate growth rate is $7/100$ or 7% per year.

Using the Harrod-Domar equation, we see that the required rate of savings is $g \times \theta$, which in this case is

\[
s = \theta(g + n) \Rightarrow s = 4\left(\frac{7}{100}\right) = \frac{28}{100} = 28\%
\]

3.c The trick in this problem is to calculate what is, effectively, the capital-output ratio in Xanadu because of the labor problems. Basically, if $\theta$ is the amount of capital you need to produce a single unit of output, you will now effectively end up using more than that. How much more? Well, it must be $\theta \times (4/3)$. If you take away a quarter of this, you will get back exactly $\theta$.  

So the effective capital-output ratio is now $4 \times (4/3) = 16/3$. Using this in the Harrod-Domar equation with a rate of savings is 1/5, we see that $g = 3/80$, which is 3.75% per year.

$$\frac{s}{\theta} = \frac{1/5}{16/3} = \frac{3}{80}$$

Subtract the population growth rate.

$$\frac{s}{\theta} = g + n \Rightarrow \frac{3}{80} - \frac{2}{100} = g \Rightarrow \frac{30 - 16}{800} = g \Rightarrow \frac{14}{800} = g$$

The answer for per-capita growth is therefore 1.75% per year.

3.d Economic well-being comes from a mix of both current consumption and future consumption. A higher savings rate benefits future consumption at the expense of current consumption. So our objective should not be to always raise savings rates, but find some intermediate rate of savings that permits a desirable combination of current and future consumption.

6) This problem will help you understand how the steady state in the Solow model is described. To solve this problem we use the functional form given,

$$Y(t) = AK(t)^{\alpha}L(t)^{1-\alpha}$$

which describes how total output is produced with capital and labor. We transform this into a per-capita magnitude by dividing through by the labor force (there is no technical progress here so that labor is just the same as effective labor). If we define $y = Y/L$ and $k = K/L$, we see that

$$y(t) = Ak^\alpha.$$ 

Second, we use the equation in the Solow model which describes the relation between future capital and current capital

$$(1 + n)k(t + 1) = (1 - \delta)k(t) + sy(t)$$

which we can rewrite using the specific functional form above as

$$(1 + n)k(t + 1) = (1 - \delta)k(t) + sAk(t)^\alpha$$

In the steady state $\tilde{k}$ (the steady state level of capital), current capital stock equals future capital stock or $k(t) = k(t + 1) = \tilde{k}$. Consequently,

$$(1 + n) \tilde{k} = (1 - \delta) \tilde{k} + sA\tilde{k}^\alpha$$

Now we solve this equation to figure out what the value of $\tilde{k}$ is,

$$\frac{\tilde{k}(n + \delta)}{\tilde{k}^\alpha} = sA \Rightarrow \frac{\tilde{k}}{\tilde{k}^\alpha} = sA \frac{1}{n + \delta} \Rightarrow \tilde{k}^{(1-\alpha)} = \frac{sA}{(n + \delta)} \Rightarrow \tilde{k} = \left(\frac{sA}{(n + \delta)}\right)^{\frac{1}{1-\alpha}}$$

Now using this equation, you should be able to easily tell the direction in which $\tilde{k}$ moves, in response to all the changes asked about in the question.

8.a) True. Here write down the Harrod-Domar equation. And then go on to mention that in the Solow model, long-run growth rate is determined simply by the exogenous rate
of technical progress. The savings rate only determines long-run capital stocks per-capita and the level of per-capita output, not its rate of growth.

8.b) False. Simply write down the Harrod-Domar equation and argue that an increase in the capital-output ratio must lower the rate of growth.

8.c) False. Studying countries that are currently rich introduces a bias towards convergence, as you are simply selecting ex post countries that were successful and so similar. You can mention Baumol’s study as an example of this kind of mistake.

8.d) True. Quah’s study of mobility of countries shows that both very poor and very rich countries are unlikely to change world rankings all that much. In contrast, countries that were middle-income in 1960 have shown remarkable changes. A large fraction of them have become dramatically richer, while a large fraction have also become dramatically poorer.

8.e) True (assuming technological progress). In the Solow model, population growth has only a level effect on long-run per-capita income. Here you may draw a quick diagram that describes the steady state in the Solow model and show what happens as population growth increases. Then point out that in the long-run, the rate of growth in the Solow model is just the rate of technical progress.

8.f) False. Draw the production function relating output per head to capital per head. Of course output per head increases as capital per head increases. The point is that it does so at a diminishing rate, but it increases nevertheless.

(10) A country with a lower ratio of capital to labor might grow faster for two reasons: (1) Its marginal product of capital may be higher because there are lots of labor to work with the capital (diminishing returns); (2) Low capital is also likely to mean certain old technologies can be more easily scrapped because they are hopelessly out of date or nonexistent to start with (phone, computer, television networks for example). This is more difficult for richer countries which have (perhaps not fully modern, but still valuable) infrastructural systems already in place. So these are two factors that bear on convergence.

Low ratios of capital to labor may also make for slower growth. Here are two reasons: 1.) A low amount of capital relative to labor makes it likely that the country is poor, and hence has also a low amount of human capital (or skilled labor). If human capital is complementary with physical capital, this will lower the marginal product of physical capital and result in slower growth. 2.) A low ratio of capital to labor may also result in historical lock-in of the sort described by Rosenstaein-Rodan and Hirschman (see Chapter 4). This will result in lower growth as well. These are factors that bear on divergence.